PROGAMME OUTCOME

- Nationalistic Outlook and contribution to National development
- Fostering global competencies, and Technical and Intellectual proficiency
- Inculcating values and Social Commitment
- Affective skills and integrity of character
- Critical Thinking, Problem solving and Research-related skills
- Environment and sustainability
- Quest for excellence.

PROGAMME SPECIFIC OUTCOME

On Successful completion of this course, students will

- get the strong base of different areas of Mathematics and to apply those ideas in other disciplines and also in daily life to a certain extent.
- develop an analytic mind and assists in better organization of ideas and accurate expression of thoughts.
- be able to understand the world around them with mathematical models of natural phenomena, of human behaviour and of social systems.
- be able to think critically

COURSE OUTCOMES

FIRST SEMESTER

MTS1 B01 BASIC LOGIC & NUMBER THEORY

Aims, Objectives and Outcomes

Logic, the study of principles of techniques and reasoning, is fundamental to every branch of learning. Besides, being the basis of all mathematical reasoning, it is required in the field of computer science for developing programming languages and also to check the correctness of the programmes. Electronic engineers apply logic in the design of computer chips. The first module discusses the fundamentals of logic, its symbols and rules. This enables one to think systematically, to express ideas in precise and concise mathematical terms and also to make valid arguments. How to use logic to arrive at the correct conclusion in the midst of confusing and contradictory statements is also illustrated.

The classical number theory is introduced and some of the very fundamental results are discussed in other modules. It is hoped that the method of writing a formal proof, using proof methods discussed in the first module, is best taught in a concrete setting, rather than as an abstract exercise in logic. Number theory, unlike other topics such as geometry and analysis, doesn't suffer from too much abstraction and the consequent difficulty in conceptual understanding. Hence, it is an ideal topic for a beginner to illustrate how mathematicians do their normal business. By the end of the course, the students will be able to enjoy and master several techniques of problem solving such as recursion, induction etc., the importance of pattern recognition in mathematics, the art of conjecturing and a few applications of number theory. Enthusiastic students will have acquired knowledge to read and enjoy on their own a few applications of number theory in the field of art, geometry and coding theory. Successful completion of the course enables students to

- Prove results involving divisibility, greatest common divisor, least common multiple and a few applications.
- Understand the theory and method of solutions of LDE.
- Solve linear congruent equations.
- Learn three classical theorems *viz*. Wilson's theorem, Fermat's little theorem and Euler's theorem and a few important consequences.

SECOND SEMESTER

Aims, Objectives and Outcomes

The mathematics required for *viewing* and analyzing the physical world around us is contained in calculus. While Algebra and Geometry provide us very useful tools for expressing the relationship between static quantities, the concepts necessary to explore the relationship between moving/changing objects are provided in calculus. The objective of the course is to introduce students to the fundamental ideas of limit, continuity and differentiability and also to some basic theorems of *differential calculus*. It is also shown how these ideas can be applied in the problem of sketching of curves and in the solution of some optimization problems of interest in real life. This is done in the first two modules.

The next two modules deal with the other branch of calculus *viz. integral calculus*. Historically, it is motivated by the geometric problem of finding out the area of a planar region. The idea of *definite integral* is defined with the notion of limit. A major result is the *Fundamental Theorem of Calculus*, which not only gives a practical way of evaluating the definite integral but establishes the close connection between the two branches of Calculus. The notion of definite integral not only solves the area problem but is useful in finding out the arc length of a plane curve, volume and surface areas of solids and so on. The integral turns out to be a powerful tool in solving problems in physics, chemistry, biology, engineering, economics and other fields. Some of the applications are included in the syllabus.

THIRD SEMESTER

MTS3 B03 CALCULUS OF SINGLE VARIABLE2

Aims, Objectives and Outcomes

Using the idea of definite integral developed in previous semester, the natural logarithm function is defined and its properties are examined. This allows us to define its inverse function namely the *natural exponential function* and also the *general exponential function*. Exponential functions model a wide variety of phenomenon of interest in science, engineering, mathematics and economics. They arise naturally when we model the growth of a biological population, the spread of a disease, the radioactive decay of atoms, and the study of heat transfer problems and so on. We also consider certain combinations of exponential functions namely *hyperbolic functions* that also arise very frequently in applications such as the study of shapes of cables hanging under their own weight.

After this, the students are introduced to the idea of *improper integrals*, their convergence and evaluation. This enables to study a related notion of convergence of a *series*, which is practically done by applying several different tests such as integral test, comparison test and so on. As a special case, a study on power series their region of convergence, differentiation and integration etc., is also done.

A detailed study of plane and space curves is then taken up. The students get the idea of parametrization of curves, they learn how to calculate the arc length, curvature etc. using parametrization and also the area of surface of revolution of a parametrized plane curve. Students are introduced into other coordinate systems which often simplify the equation of curves and surfaces and the relationship between various coordinate systems are also taught. This enables them to directly calculate the arc length and surface areas of revolution of a curve whose equation is in polar form. At the end of the course, the students will be able to handle *vectors* in dealing with the problems involving geometry of lines, curves, planes and surfaces in space and have acquired the ability to sketch curves in plane and space given in vector valued form.

FOURTH SEMESTER

MTS4 B04 LINEAR ALGEBRA

Aims, Objectives and Outcomes

An introductory treatment of linear algebra with an aim to present the fundamentals in the clearest possible way is intended here. Linear algebra is the study of linear systems of equations, vector spaces, and linear transformations. Virtually every area of mathematics relies on or extends the tools of linear algebra. Solving systems of linear equations is a basic tool of many mathematical procedures used for solving problems in science and engineering. A number of methods for solving a system of linear equations are discussed. In this process, the student will become competent to perform matrix algebra and also to calculate the inverse and determinant of a matrix. Another advantage is that the student will come to understand the modern view of a matrix as a linear transformation. The discussion necessitates the introduction of central topic of linear algebra namely the concept of a *vector space*. The familiarity of the students with planar vectors and their algebraic properties under vector addition and scalar multiplication will make them realize that the idea of a general vector space is in fact an *abstraction* of what they already know. Several examples and general properties of vector spaces are studied. The idea of a subspace, spanning vectors, basis and dimension are discussed and fundamental results in these areas are explored. This enables the student to understand the relationship among the solutions of a given system of linear equations and some important subspaces associated with the coefficient matrix of the system.

After this, some basic matrix transformations in the vector spaces and , having interest in the field of computer graphics, engineering and physics are studied by specially pinpointing to their geometric effect.

Just like choosing an appropriate coordinate system greatly simplifies a problem at our hand as we usually see in analytic geometry and calculus, a right choice of the basis of the vector space greatly simplifies the analysis of a matrix operator on it. With this aim in mind, a study on eigenvalues and eigenvectors of a given matrix (*equivalently*, that of the corresponding matrix operator) is taken up. Practical method of finding out the eigenvalues from the characteristic equation and the corresponding eigenvectors are also discussed. A bonus point achieved during this process is a test for the invertibility of a square matrix. As diagonal matrices are the matrices with simplest structure, the idea of *diagonalization* of a matrix (and hence the diagonalization of a matrix operator) is introduced and students learn a few fundamental results involving diagonalization and eigenvalues which enable them to check whether diagonalization is possible. They realise that there are matrices that cannot be diagonalized and even learn to check it. Also they are taught a well defined procedure for diagonalizing a given matrix, if this is actually the case. The topic is progressed further to obtain the ultimate goal of *spectral decomposition* of a symmetric matrix. In this process , students realise that every symmetric matrix is diagonalizable and that this diagonalization can be done in a special way ie., by choosing an *orthogonal matrix* to perform the diagonalization. This is known as orthogonal diagonalization. Students also learn that *only* symmetric matrices with real entries can be orthogonally diagonalized and using Gram Schmidt process a well defined procedure for writing such a diagonalization is also taught. In short, the course gives the students an opportunity to learn the fundamentals of linear algebra by capturing the ideas geometrically, by justifying them algebraically and by preparing them to apply it in several different fields such as data communication, computer graphics, modelling etc.

MTS5 B05 ABSTRACT ALGEBRA

Aims, Objectives and Outcomes

The brilliant mathematician Evariste Galois developed an entire theory that connected the solvability by radicals of a polynomial equation with the *permutation group* of its roots. The theory now known as *Galois theory* solves the famous problem of *insolvability of quintic.* A study on *symmetric functions* now becomes inevitable. One can now observe the connection emerging between classical algebra and modern algebra. The last three modules are therefore devoted to the discussion on basic ideas and results of abstract algebra. Students understand the abstract notion of a group, learn several examples, are taught to check whether an *algebraic system* forms a group or not and are introduced to some fundamental results of group theory. The idea of structural similarity, the notion of cyclic group, permutation group , various examples and very fundamental results in the areas are also explored.

MTS5 B06 BASIC ANALYSIS

Aims, Objectives and Outcomes

In this course, basic ideas and methods of real and complex analysis are taught. Real analysis is a theoretical version of single variable calculus. So many familiar concepts of calculus are reintroduced but at a much deeper and more rigorous level than in a calculus course. At the same time there are concepts and results that are new and not studied in the calculus course but very much needed in more advanced courses. The aim is to provide students with a level of mathematical sophistication that will prepare them for further work in mathematical analysis and other fields of knowledge, and also to develop their ability to analyse and prove statements of mathematics using logical arguments. The course will enable the students

- to learn and deduce rigorously many properties of real number system by assuming a few fundamental facts about it as axioms. In particular they will learn to prove Archimedean property, density theorem, existence of a positive square root for positive numbers and so on and the learning will help them to appreciate the beauty of logical arguments and embolden them to apply it in similar and unknown problems.

- to know about sequences ,their limits, several basic and important theorems involving sequences and their applications . For example, they will learn how *monotone convergence theorem* can be used in establishing the divergence of the *harmonic series*, how it helps in the calculation of square root of positive numbers and how it establishes the existence of the *transcendental* number *e (Euler constant).*

- to understand some basic topological properties of real number system such as the concept of open and closed sets, their properties, their characterization and so on.

- to get a rigorous introduction to algebraic, geometric and topological structures of complex number system, functions of complex variable, their limit and continuity and so on. Rich use of geometry, comparison between real and complex calculus-areas where they agree and where they differ, the study of mapping properties of a few important complex functions exploring the underlying geometry etc. will demystify student’s belief that complex variable theory is incomprehensible.

FIFTH SEMESTER

MTS5 B07 NUMERICAL ANALYSIS

The goal of numerical analysis is to provide techniques and algorithms to find *approximate numerical solution* to problems in several areas of mathematics where it is impossible or hard to find the actual/closed form solution by analytical methods and also to make an *error analysis* to ascertain the accuracy of the *approximate solution*. The subject addresses a variety of questions ranging from the approximation of functions and integrals to the approximate solution of algebraic, transcendental, differential and integral equations, with particular emphasis on the stability, accuracy, efficiency and reliability of numerical algorithms. The course enables the students to

- Understand several methods such as bisection method, fixed point iteration method, regula falsi method etc. to find out the approximate numerical solutions of algebraic and transcendental equations with desired accuracy.
- Understand the concept of interpolation and also learn some well known interpolation techniques.
- Understand a few techniques for numerical differentiation and integration and also realize their merits and demerits.

Find out numerical approximations to solutions of initial value problems and also to understand the efficiency of various methods.

FIFTH SEMESTER

MTS5 B08 LINEAR PROGRAMMING

Aims, Objectives and Outcomes

Linear programming problems are having wide applications in mathematics, statistics, computer science, economics, and in many social and managerial sciences. For mathematicians it is a sort of mathematical modelling process, for statisticians and economists it is useful for planning many economic activities such as transport of raw materials and finished products from one place to another with minimum cost and for military heads it is useful for scheduling the training activities and deployment of army personnel. The emphasis of this course is on nurturing the linear programming skills of students *via*. the algorithmic solution of smallscale problems, both in the general sense and in the specific applications where these problems naturally occur. On successful completion of this course, the students will be able to

- solve linear programming problems geometrically
- understand the drawbacks of geometric methods
- solve LP problems more effectively using Simplex algorithm *via*. the use of condensed tableau of A.W. Tucker
- convert certain related problems, not *directly* solvable by simplex method, into a form that can be attacked by simplex method.
- understand duality theory, a theory that establishes relationships between linear programming problems of maximization and minimization
- understand game theory
- solve transportation and assignment problems by algorithms that take advantage of the simpler nature of these problems

MTS5 B09 INTRODUCTION TO GEOMETRY AND THEORY OF EQUATIONS

Geometry

Geometry is, basically, the study concerned with questions of shape, size, and relative position of planar and spatial objects. The classical Greek geometry, also known as *Euclidean geometry* after the work of Euclid, was once regarded as one of the highest points of rational thought, contributing to the thinking skills of logic, deductive reasoning and skills in problem solving.

In the early 17 th century, the works of Rene Descartes and Pierre de Fermat put the foundation stones for the creation of *analytic geometry* where the idea of a coordinate system was introduced to simplify the treatment of geometry and to solve a wide variety of geometric problems.

Desargues, a contemporary of Descartes was fascinated towards the efforts of artists/painters to give a realistic view of their art works/paintings usually done on a flat surface such as canvas or paper. To get a realistic view of a three dimensional object/scene depicted on a flat surface, a right impression of height, width, depth and position in relation to each other of the objects in the scene is required. This idea is called *perspective* in art. If two artists make perspective drawings of the same object, their drawings shall not be identical but there shall be certain properties of these drawings that remain the same or that remain *invariant*. The study of such invariant things crystallised into what is now called *projective geometry*. Now days, it plays a major role in computer graphics and in the design of camera models.

Another development is the evolution of *affine geometry.* In simple terms, if we look at the shadows of a rectangular window on the floor under sunlight, we could see the shadows not in perfect rectangular form but often in the shape of a parallelogram. The size of shadows also changes with respect to the position of the sun. Hence, neither length nor angle is *invariant* in the *transformation* process. However, the opposite sides of the images are always parallel. So this transformation keeps *parallelism* intact. The investigation of *invariants* of *all* shadows is the basic problem of affine geometry.

Towards the end of nineteenth century, there were several different geometries: Euclidean, affine, projective, inversive, spherical, hyperbolic, and elliptic to name a few. It was the idea of Felix Klein to bring the study of all these different geometries into a single platform. He viewed each geometry as a space together with a *group of transformations* of that space and regarded those properties of figures left unaltered by the group as geometrical properties. In this course, it is intended to take up a study of a few geometries based on the *philosophy* of Klein.

Theory of equations

Theory of equations is an important part of traditional algebra course and it mainly deals with polynomial equations and methods of finding their *algebraic solution* or *solution by radicals*. This means we seek a formula for solutions of polynomial equations in terms of coefficients of polynomials that involves only the operations of addition, subtraction, multiplication, division and taking roots. A well knitted formula for the solution of a quadratic polynomial equation is known to us from high school classes and is not difficult to derive. However, there is an increasing difficulty to derive such a formula for polynomial equations of third and fourth degree. One of our tasks in this learning process is to derive formulae for the solutions of *third* and *fourth* degree polynomial equations given by Carden and Ferrari respectively. In the mean time, we shall find out the relationship between the roots and coefficients of an n^{th} degree polynomial and an upper and lower limit for the roots of such a polynomial. This often help us to locate the region of solutions for a general polynomial equation. Methods to find out integral and rational roots of a general n^{th} degree polynomial with rational coefficients are also devised. However, all efforts to find out an *algebraic solution* for general polynomial equations of degree higher than fourth failed or didn't work. This was not because one failed to hit upon the right idea, but rather due to the disturbing fact that there was no such formula.

Upon successful completion of the course, students will be able to

- Understand several basic facts about parabola, hyperbola and ellipse *(conics)* such as their equation in standard form, focal length properties, and reflection properties, their tangents and normal.
- Recognise and classify conics.
- Understand Kleinian view of Euclidean geometry.
- Understand affine transformations, the inherent group structure, the idea of parallel projections and the basic properties of parallel projections.
- Understand the fundamental theorem of affine geometry.
- Learn to solve polynomial equations upto degree four.

SIXTH SEMESTER

MTS6 B10 REAL ANALYSIS

Aims, Objectives and Outcomes

The course is built upon the foundation laid in Basic Analysis course of fifth semester. The course thoroughly exposes one to the rigour and methods of an analysis course. One has to understand definitions and theorems of text and study examples well to acquire skills in various problem solving techniques. The course will teach one how to combine different definitions, theorems and techniques to solve problems one has never seen before. One shall acquire ability to realise when and how to apply a particular theorem and how to avoid common errors and pitfalls. The course will prepare students to formulate and present the ideas of mathematics and to communicate them elegantly.

On successful completion of the course, students will be able to

- State the definition of continuous functions, formulate sequential criteria for continuity and prove or disprove continuity of functions using this criteria.
- Understand several deep and fundamental results of continuous functions on intervals such as boundedness theorem, maximumminimum theorem, intermediate value theorem, preservation of interval theorem and so on.
- Realise the difference between continuity and uniform continuity and equivalence of these ideas for functions on closed and bounded interval.
- Understand the significance of uniform continuity in continuous extension theorem.
- Develop the notion of Riemann integrability of a function using the idea of tagged partitions and calculate the integral value of some simple functions using the definition.
- Understand a few basic and fundamental results of integration theory.
- Formulate Cauchy criteria for integrability and a few applications of it. In particular they learn to use Cauchy criteria in proving the non integrability of certain functions.
- Understand classes of functions that are always integrable
- Understand two forms of fundamental theorem of calculus and their significance in the practical problem of evaluation of an integral.
- Find a justification for ‘change of variable formula’ used in the practical problem of evaluation of an integral.
- Prove convergence and divergence of sequences of functions and series

- Understand the difference between pointwise and uniform convergence of sequences and series of functions
- Answer a few questions related to interchange of limits.
- Learn and find out examples/counter examples to prove or disprove the validity of several mathematical statements that arise naturally in the process/context of learning.
- Understand the notion of improper integrals, their convergence, principal value and evaluation.
- Learn the properties of and relationship among two important improper integrals namely *beta and gamma functions* that frequently appear in mathematics, statistics, science and engineering.

MTS6 B11 COMPLEX ANALYSIS

Aims, Objectives and Outcomes

The course is aimed to provide a thorough understanding of complex function theory. It is intended to develop the topics in a fashion analogous to the calculus of real functions. At the same time differences in both theories are clearly emphasised. When real numbers are replaced by complex numbers in the definition of derivative of a function, the resulting *complex differentiable functions* (more precisely *analytic functions)* turn out to have many remarkable properties not possessed by their real analogues. These functions have numerous applications in several areas of mathematics such as differential equations, number theory etc. and also in science and engineering. The focus of the course is on the study of analytic functions and their basic behaviour with respect to the theory of complex calculus.

The course enables students

- to understand the difference between differentiability and analyticity of a complex function and construct examples.
- to understand necessary and sufficient condition for checking analyticity.
- to know of harmonic functions and their connection with analytic functions
- to know a few elementary analytic functions of complex analysis and their properties.
- to understand definition of complex integral, its properties and evaluation.
- to know a few fundamental results on contour integration theory such as Cauchy's theorem, Cauchy-Goursat theorem and their applications.
- to understand and apply Cauchy's integral formula and a few consequences of it such as Liouville's theorem, Morera's theorem and so forth in various situations.
- to see the application of Cauchy's integral formula in the derivation of power series expansion of an analytic function.
- to know a more general type of series expansion analogous to power series expansion *viz.* *Laurent's series expansion* for functions having *singularity*.
- to understand how Laurent's series expansion lead to the concept of *residue,* which in turn provide another fruitful way to evaluate complex integrals and, in some cases, even real integrals.
- to see another application of residue theory in locating the region of zeros of an analytic function.

SIXTH SEMESTER

MTS6 B12 CALCULUS OF MULTI VARIABLE

Aims, Objectives and Outcomes

The intention of the course is to extend the immensely useful ideas and notions such as limit, continuity, derivative and integral seen in the context of function of single variable to function of several variables. The corresponding results will be the higher dimensional analogues of what we learned in the case of single variable functions. The results we develop in the course of calculus of multivariable is extremely useful in several areas of science and technology as many functions that arise in real life situations are functions of multivariable.

The successful completion of the course will enable the student to

- Understand several contexts of appearance of multivariable functions and their representation using graph and contour diagrams.
- Formulate and work on the idea of limit and continuity for functions of several variables.
- Understand the notion of *partial derivative*, their computation and interpretation.
- Understand chain rule for calculating partial derivatives.
- Get the idea of *directional derivative*, its evaluation, interpretation, *and* relationship with partial derivatives.
- Understand the concept of *gradient*, a few of its properties, application and interpretation.
- Understand the use of partial derivatives in getting information of tangent plane and normal line.
- Calculate the maximum and minimum values of a multivariable function using second derivative test and Lagrange multiplier method.
- Find a few real life applications of Lagrange multiplier method in optimization problems.
- Extend the notion of integral of a function of single variable to integral of functions of two and three variables.
- Address the practical problem of evaluation of double and triple integral using Fubini's theorem and change of variable formula.
- Realise the advantage of choosing other coordinate systems such as polar, spherical, cylindrical etc. in the evaluation of double and triple integrals .
- See a few applications of double and triple integral in the problem of finding out surface area ,mass of lamina, volume, centre of mass and so on.
- Understand the notion of a vector field, the idea of curl and divergence of a vector field, their evaluation and interpretation.
- Understand the idea of line integral and surface integral and their evaluations.
- Learn three major results viz. Green's theorem, Gauss's theorem and Stokes' theorem of multivariable calculus and their use in several areas and directions.

SIXTH SEMESTER

MTS6 B13 DIFFERENTIAL EQUATIONS

Aims, Objectives and Outcomes

Differential equations model the physical world around us. Many of the laws or principles governing natural phenomenon are statements or relations involving rate at which one quantity changes with respect to another. The mathematical formulation of such relations (*modelling*) often results in an equation involving derivative (*differential equations*). The course is intended to find out ways and means for solving differential equations and the topic has wide range of applications in physics, chemistry, biology, medicine, economics and engineering.

On successful completion of the course, the students shall acquire the following skills/knowledge.

- Students could identify a number of areas where the modelling process results in a differential equation.
- They will learn what an ODE is, what it means by its solution, how to classify DEs, what it means by an IVP and so on.
- They will learn to solve DEs that are in linear, separable and in exact forms and also to analyse the solution.
- They will realise the basic differences between linear and non linear DEs and also basic results that guarantees a solution in each case.
- They will learn a method to approximate the solution successively of a first order IVP.
- They will become familiar with the theory and method of solving a second order linear homogeneous and nonhomogeneous equation with constant coefficients.
- They will learn to find out a *series solution* for homogeneous equations with variable coefficients near *ordinary points*.
- Students acquire the knowledge of solving a differential equation using Laplace method which is especially suitable to deal with problems arising in engineering field.

Students learn the technique of solving *partial differential equations* using the method of separation of variables.